Fuzzy Set Theory in Computer Vision:
Example 12

Derek T. Anderson and James M. Keller

FUZZ-IEEE, July 2017

Overview

- Already discussed how to learn μ from data

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$
- These are called the densities

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$
- These are called the densities
- They are the utility of the individuals

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$
- These are called the densities
- They are the utility of the individuals
- Maybe they were given to us

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$
- These are called the densities
- They are the utility of the individuals
- Maybe they were given to us
- Maybe we are trying to learn them from data

Overview

- Already discussed how to learn μ from data
- Sometimes, all we know is $\mu(x_i), \forall i \in \{1, ..., N\}$
- These are called the densities
- They are the utility of the individuals
- Maybe they were given to us
- Maybe we are trying to learn them from data
- The focus of these slides are how to "fill in" (aka impute) the remaining terms, i.e., $\mu(A), \forall A \in 2^X \setminus \{x_1, ..., x_N\}$

Sugeno λ FM

- For sets $A, B \subseteq X$, such that $A \cap B = \emptyset$,

$$\mu_\lambda(A \cup B) = \mu_\lambda(A) + \mu_\lambda(B) + \lambda\mu_\lambda(A)\mu_\lambda(B),$$

for some $\lambda > -1$.

Sugeno λ FM

- For sets $A, B \subseteq X$, such that $A \cap B = \emptyset$,

$$\mu_\lambda(A \cup B) = \mu_\lambda(A) + \mu_\lambda(B) + \lambda \mu_\lambda(A)\mu_\lambda(B),$$

  for some $\lambda > -1$.

- Sugeno showed,

$$\lambda + 1 = \prod_{i=1}^{N} \left(1 + \lambda \mu^i\right),$$

  there exists exactly one real solution such that $\lambda > -1$, where $\mu^i = \mu(x_i)$.

Interesting observations

- If $\sum_{i=1}^{N} \mu^i = 1$, then $\lambda = 0$ (probability measure)

Interesting observations

- If $\sum_{i=1}^{N} \mu^i = 1$, then $\lambda = 0$ (probability measure)
- If $\sum_{i=1}^{N} \mu^i > 1$, then $\lambda < 0$ (Dempster-Shafer belief function)

Interesting observations

- If $\sum_{i=1}^{N} \mu^i = 1$, then $\lambda = 0$ (probability measure)
- If $\sum_{i=1}^{N} \mu^i > 1$, then $\lambda < 0$ (Dempster-Shafer belief function)
- If $\sum_{i=1}^{N} \mu^i < 1$, then $\lambda > 0$ (Dempster-Shafer plausibility function)

S-decomposable measure

- Let S be a t-conorm

S-decomposable measure

- Let S be a t-conorm
- Calculate
$$\mu(A \cup B) = S\left(\mu(A), \mu(B)\right).$$

S-decomposable measure

- Let S be a t-conorm
- Calculate

$$\mu(A \cup B) = S\left(\mu(A), \mu(B)\right).$$

- Famous example is the W^*-decomposable measure, where W^* is the Lukasiewicz t-norm

S-decomposable measure

- Let S be a t-conorm
- Calculate

$$\mu(A \cup B) = S\left(\mu(A), \mu(B)\right).$$

- Famous example is the W^*-decomposable measure, where W^* is the Lukasiewicz t-norm
- For example, let S be the maximum operator

$$\mu(A \cup B) = max(\mu(A), \mu(B)).$$

Thus, if we only have the densities then the utility of any subset is defined to be the "strongest" (highest utility) individual in that subset.